

Mathematical Modelling of the Population Dynamics of Two- Prey and One- Predator Systems at the Human-Livestock-Wildlife Interface

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Abstract

Understanding the population dynamics at the human-livestock-wildlife interface is key to managing zoonotic and cross-species diseases as well as maintaining ecosystem biodiversity at this interface. This necessitates elucidation of the effects of within and between species interactions and human activities such as farming and animal harvesting among others.

In this study, a mathematical model was developed and analyzed to study the dynamics of two- prey (Uganda-kobs (*kobus kob thomasi*) and the buffaloes (*Syncerus caffer*)) and one-predator (the lions (*Panthera leo*)) system at the human-livestock-wildlife interface. The model was analysed qualitatively for equilibrium points and their stability and, upon parametrization based on data in literature, numerical simulations were performed.

Our findings re-echoed/re-emphasized that, for co-existence of the three species, the rate of human harvesting of *kobus kob thomasi* needed to be maintained below the species' intrinsic growth rate. Existence of such a critical harvesting threshold was demonstrated and any harvesting rate exceeding that threshold would lead to ultimate extinction of both the *kobus kob thomasi* and the *Panthera leo*. Our findings further revealed that, for their long-term survival, the predator needs not to focus only on the easy-to-catch prey but use a balanced approach to ensure continued survival of both prey species.

We conclude that, given the field data limitations, our findings are rather preliminary and more of a basis for future studies geared towards improving management of ecosystems involving interacting species. Most importantly, this study demonstrates that mathematical models can play a significant role in tackling complex system dynamics to generate useful information to guide policy decisions.

Keywords: Prey-Predator Systems; Predation; Population Dynamics; Ecological Modelling; Harvesting Threshold

Introduction

Gaining insight into the rather complex population dynamics at the human-livestock-wildlife interface e.g. around game parks and zoos is an important component in the efforts to minimize the impact of zoonotic and cross-species diseases such as African swine fever and foot-and-mouth disease. Besides this factor, it is also a primary aim of environmentalists and biological conservationists to preserve

a rich environment biodiversity with species co-existence in an ecosystem at this interface. However, the species ecology at such an interface is so complex to understand [1,2] yet this knowledge is paramount for better intervention. The population dynamics in wildlife species are often driven by ecological processes that are often difficult to elucidate. Mathematical models such as disease-dynamic models have the ability to integrate and evaluate multiple, complex processes simultaneously in an efficient manner [2-6] and are hence handy in understanding biological phenomena. Mathematical modelling refers to the use of mathematical methods to analyze and solve problems that arise from real world systems that may include physical, financial, social, and ecological systems among others. They involve the use of theory of linear and non-linear differential equations and these have long played an important role in the field of theoretical population dynamics. Differential equation (both partial and ordinary) models for interactions between species are one of the classical applications of mathematics to biology [7].

Mathematical ecology involves the study of populations that interact, thereby affecting each other's growth rates. Species-interactions exist throughout nature and can be characterized by competition, cooperation, predation, parasitism, mutualism and commensalism [8] among others. Mathematical population models have been used to study the dynamics of prey-predator systems since Lotka [9] and Volterra [10] in Italy independently proposed their model. Since then, many mathematical models have been developed based on more implicit biological assumptions. Examples include the Malthusian growth model [11,12], the logistic growth model [13] and cell population model [14] among others. When modelling prey-predator dynamics, incorporation of the predator rate of capture, the handling time of catching and consuming a prey, and predator satiation [15,16] is key. In nature, the rate of harvest by a predator population is determined by two attributes of the predator's response to changes in prey density, namely; functional response and numerical response [15,17-19]. In population dynamics and ecology, functional response defines the changes in the per capita predation rate as prey density increases and it describes its per capita feeding rate on prey [20-22]. Holling [20] suggested that the predator should not be able to consume an unlimited number of prey as the prey population increases.

Practically, how predators respond to changes in prey availability (functional response) is an issue of particular importance. There is evidence from several studies that the type of functional response specified can greatly affect model predictions [23]. Generally, three general forms of functional response are commonly used in ecological models namely the linear, hyperbolic and sigmoidal form [24,25] and are generally classified into three types, namely Holling Type I, II and III [26]. The effect of interspecies competition too has been studied previously [27] and we incorporate that in this study.

Here, mathematical models were used to investigate the effect of interspecies competition among preys, human harvesting rate of prey and predation/capture rates of the preys by predator on the population dynamics in a two- prey (i.e. Uganda-kob (*kobus kob thomasi*) and buffaloes (*Syncerus caffer*) and one-predator (i.e. lion (*Panthera leo*) ecosystem at the human-livestock-wildlife interface to cater for human harvesting.

Materials and Methods

Model building

This study involved adaptation of models from closely related studies [7,17,28,29] that also incorporated both intra- and inter-species competition among prey and predators into the dynamics. Here, we incorporated functional responses, Holling Type II, ratio-dependent response of the predator, constant effort harvesting (harvesting) of the prey [30,31] and logistic growth of the prey species in the absence of the predators. The model was parameterized based on existing literature and expert opinion where needed.

Model assumptions

The following assumptions are made in building this study's model:

1. No other factors like rainfall and diseases affect the dynamics of this system.
2. *kobus kob thomasi* is easy to capture by the lion and capturing buffalo requires a lot of searching time for the lion.

3. The rate of harvesting of the *kobus kob thomasi* is on average constant per unit time and so it is represented as constant effort harvesting of the prey.
4. There is logistic growth of prey in the absence of predation or harvesting, it grows until it reaches the carrying capacity.
5. The predator's growth depends on the prey it catches, hence in the absence of the prey, the predator dies out exponentially.

Model variables and parameters

The following variables are used in the model:

1. $X(t)$ is the population size of the Uganda-kob (*kobus kob thomasi*) at time t , simplified to X .
2. $Y(t)$ is the population size of the buffaloes (*Syncerus caffer*) at time t , simplified to Y .
3. $Z(t)$ is the population size of the lions (*Panthera leo*) at time t , simplified to Z .

The following are the parameters (with all rates having units of per year) used in the model:

1. r_1 and r_2 are per capita intrinsic growth rates for prey and respectively.
2. K_1 and K_2 are carrying capacities for prey and respectively.
3. α_1 and α_2 are coefficients for interspecies competition between preys and respectively.
4. a_1 and c are capturing rates of predator on respectively.
5. b_1 measures the effect of anti-predator behavior of prey.
6. h is the harvesting rate of prey.
7. e is natural mortality rate of predator.
8. d_2 measures the effect of anti-predator behavior of prey .
9. μ_1 and μ_2 are coefficients which measure the predator's efficiency to convert prey biomass of respectively into fertility (reproductivity).

Model equations

Using the above description, assumptions, variables and parameters, the proposed model of this study becomes a system of three non-linear ordinary differential equations describing how the population densities of the species would vary with time. It is represented as:

$$\frac{dX}{dt} = r_1X \left(1 - \frac{X}{K_1}\right) - \alpha_1XY - \left(\frac{a_1X}{1 + b_1X + d_2Z}\right)Z - hX \quad (1)$$

$$\frac{dY}{dt} = r_2Y \left(1 - \frac{Y}{K_2}\right) - \alpha_2XY - \left(\frac{cY}{1 + d_1Y + d_2Z}\right)Z \quad (2)$$

$$\frac{dZ}{dt} = \mu_1 \left(\frac{a_1X}{1 + b_1X + d_2Z}\right)Z + \mu_2 \left(\frac{cY}{1 + d_1Y + d_2Z}\right)Z - eZ \quad (3)$$

where all the parameters are non-negative.

Parameter estimation

Model variables and parameter values were derived from literature whenever available and if missing, expert opinion was sought. A few of the parameters were assumed for purposes of this study and we emphasize the need for future studies to generate quantitative information on the missing parameters. A description of these parameters and variables is presented in table 1 and their derivation is described under the field "source".

For example, intrinsic growth rates are derived from the net difference between the recruitment rate and the loss rate while the coefficient of interspecies competition must be derived while accounting for the differences between competing species in amount of resources used and overlap in the set of resources used. The two competition coefficients are not necessarily equal and competition coefficients are generally less than one [32]. The biomass conversion factors denote the number of newly born predators for each captured prey respectively [7]. We incorporated anti-predator parameters into the model since those behaviors can influence the predation process by, for example, decreasing rates of encounter, probabilities of detection, and probabilities of escape after detection among others [33].

Model analysis

Qualitative analysis: Although deep qualitative analyses of the model were performed, we considered them to be beyond the scope of the paper due to the level of mathematics details involved. Consequently, we only generally mention the key analyses performed. Readers interested in the in-depth qualitative analyses can contact any of the authors. Briefly, we analyzed the system in Equations (1-3) to infer the existence of non-negative equilibrium points and the conditions for their stability. By solving Equations (1-3), we obtained equilibrium solutions comprising of populations of the prey and the predator at equilibrium. We then established the conditions for their existence and stability. The local asymptotic stability of each equilibrium point was studied by computing the Jacobian matrix and finding the eigenvalues at each equilibrium point. For stability of the equilibrium points, the real parts of the eigenvalues of the Jacobian matrix must be negative. We followed well documented approaches for these analyses. See [7,17,28,29,34,35] for examples on the approaches used.

Quantitative/Numerical analysis

To investigate the dynamics of the system under different parameter conditions and to illustrate some numerical results, quantitative analyses were performed through simulations of the system in Equations (1-3). Unless otherwise stated, the parameters and variables (in table 1) were used in the simulation exercises. The dynamics of the system were simulated for a period of 100 years. The system of Equations (1-3) parametrized by the values in table 1 was solved using function in Mathematica 12.0 (Wolfram Research, Inc.) and the Code is available on request.

Parameter/variable	Description	Value	Source
	Per capita intrinsic growth rate for <i>kobus kob thomasi</i>	0.16	Estimated from birth rate of 0.33 y ⁻¹ with adult females at 58.4% [36] and life span of 17 years [37]
	Per capita intrinsic growth rate for <i>Syncerus caffer</i>	0.17	Estimated from birth rate of 0.5 y ⁻¹ and herd composition of ≈ 15/40 adult females [38] and life span of 18 years [39]
	Ecosystem’s carrying capacity for <i>kobus kob thomasi</i>	40	From maximum number reported per group https://en.wikipedia.org/wiki/Kob
	Ecosystem’s carrying capacity for <i>Syncerus caffer</i>	500	Maximum number in Huffman, 2017 [39]
	Effect, on <i>kobus kob thomasi</i> , of competition with one <i>Syncerus caffer</i>	0.1	Assumed based on (Kar and Batabyal, 2010) [7] and bearing in mind that according to (Anonymous, na-b) [32]
	Effect, on <i>Syncerus caffer</i> , of competition with one <i>kobus kob thomasi</i>	0.09	Assumed based on (Kar and Batabyal, 2010) bearing in mind that according to (Anonymous, na-b) [32]
	Capture rate of <i>kobus kob thomasi</i> by the predator <i>Panthera leo</i>	0.24	Assumed based on (Kar and Batabyal, 2010) [7]
	Measures the effect of anti-predator behavior of <i>kobus kob thomasi</i>	10	Assumed based on (Matsuda et al., 1993) [33]
	Harvesting rate of <i>kobus kob thomasi</i> by humans	0.0125	Computed from level of enforcement 0.3 x 1/24 poaching incidence in (Mayaka et al., 2004) [40]
	Measures the effect of anti-predator behavior of <i>Syncerus caffer</i>	20	Assumed based on (Matsuda et al., 1993) [33] and considering that they survive better than X.
	Interference coefficient of predator <i>Panthera leo</i>	0.24	Assumed based on (Chakraborty et al., 2015) [41]
	Capture rate of <i>Syncerus caffer</i> by the predator <i>Panthera leo</i>	0.21	Assumed based on (Kar and Batabyal, 2010) [7]
	Coefficient that measures the predator’s (<i>Panthera leo</i> ’s) efficiency to convert <i>kobus kob thomasi</i> biomass into fertility (reproductivity) i.e., food conversion efficiency	0.75	(Hatton et al., 2015) [42]
	Coefficient that measures the predator’s (<i>Panthera leo</i> ’s) efficiency to convert <i>Syncerus caffer</i> biomass into fertility (reproductivity) i.e., food conversion efficiency	0.75	(Hatton et al., 2015) [42]
	Natural mortality rate of predator (<i>Panthera leo</i>)	0.07	From life span of 15 years (Anonymous, na-a) [18]
	Initial population sizes of the <i>kobus kob thomasi</i> , <i>Syncerus caffer</i> and <i>Panthera leo</i> respectively	(40, 50, 40)	Kob (https://en.wikipedia.org/wiki/Kob), Buffaloes (Huffman, 2017) [39], Lions (Bradford, 2014b) [43]

Table 1: Description, values and sources of the parameters and variables obtained from literature and used in the simulation (all rates have units of per year).

The main objective of this section was to show numerically how the three species can co-exist in the form of steady state solution in response to changes in harvesting rate, capture/predation rate and species competition. For the harvesting rate, we assessed the dynamics of the system when the harvesting rate was set at $h = 0, 0.04, 0.08, 0.12$ and 0.16 . For the capture/predation rate, we assessed the effect of increasing the capture rate of one prey while reducing that of the other and vice versa (by either doubling or halving the respective rates) and scenarios explored involved the following parameter pairs; $(c = 0.21; a_1 = 0.24)$, $(c = 0.105, a_1 = 0.48)$, and $(c = 0.42, a_1 = 0.12)$. Lastly, for prey interspecies competition, we assessed the dynamics of the system under varying coefficients obtained by either dividing the default value by 10 or multiplying it by 10. The explored scenarios involved using the following coefficient pairs; $(\alpha_1 = 0.0001, \alpha_2 = 0.00005)$, $(\alpha_1 = 0.00001, \alpha_2 = 0.000005)$, and $(\alpha_1 = 0.001, \alpha_2 = 0.0005)$, with other parameters fixed at their default values.

Results

Qualitative analysis results: From the stability analysis, we found that, for local asymptotic stability of the equilibrium point in which only the *kobus kob thomasi* were to remain in the ecosystem after a long time, the harvesting rate of *kobus kob thomasi* must be less than their intrinsic growth rate. In addition, the rate at which the predator converts the biomass of *kobus kob thomasi* into “reproductivity” must be less than the predator’s natural mortality rate and the time it takes to handle that prey. In the absence of the *Syncerus caffer* and *Panthera leo* species, the population of *kobus kob thomasi* is globally stable provided that its intrinsic per capita growth rate is more than the rate at which they are harvested by humans. On the other hand, for global stability of the two prey species in absence of the predator, the rate of harvesting of *kobus kob thomasi* must be less than their intrinsic per capita growth rate and the ratio of the per capita growth rate of one species to its carrying capacity should be greater than the effect of interspecies competition caused by the other prey. This implies that, in the case where interspecific competition among the prey species is negligible, those species can co-exist even when the *kobus kob thomasi* are harvested, provided that such harvesting does not exceed their per capita intrinsic growth rate.

Quantitative analysis results

The results from the quantitative analysis depicting the effect of varying selected model parameters on the population dynamics of the three species are presented graphically in figures 1-3. Figure 1 presents the findings on the effect of varying the human harvesting rate of *kobus kob thomasi*, figure 2 depicts the findings on the effect of varying the prey capture rate, and figure 3 shows the findings on the effect of varying the interspecies competition among the preys. We observe in figure 1a that there exists a critical per capita harvesting threshold above which *kobus kob thomasi* species will go extinct during the 100 years. For the explored range of our model parameters, this value was shown to be around 0.04 per animal per year. For harvesting rates below this threshold and with other parameters kept at their default values, all the three species were able to co-exist in the ecosystem over the simulated period. However, for the higher harvesting rates explored, the *kobus kob thomasi* species go extinct, with time of extinction being earliest (i.e. at around 20 years) at harvesting rate of 0.16 per animal per year and latest (i.e. at around 45 years) for a rate of 0.08. For all the explored harvesting rates, the population of *Syncerus caffer* species increased towards the system’s carrying capacity albeit at varying rates that increased with increasing harvesting rates of *kobus kob thomasi* (Figure 1b). The trend observed in the *kobus kob thomasi* population is clearly reflected in the decline rates of the *Panthera leo* population in figure 1c, with their population declining slowly for lower *kobus kob thomasi* harvesting rates and vice versa. Most importantly, at harvesting rates below the critical threshold, all the three species co-existed throughout the 100 years’ period.

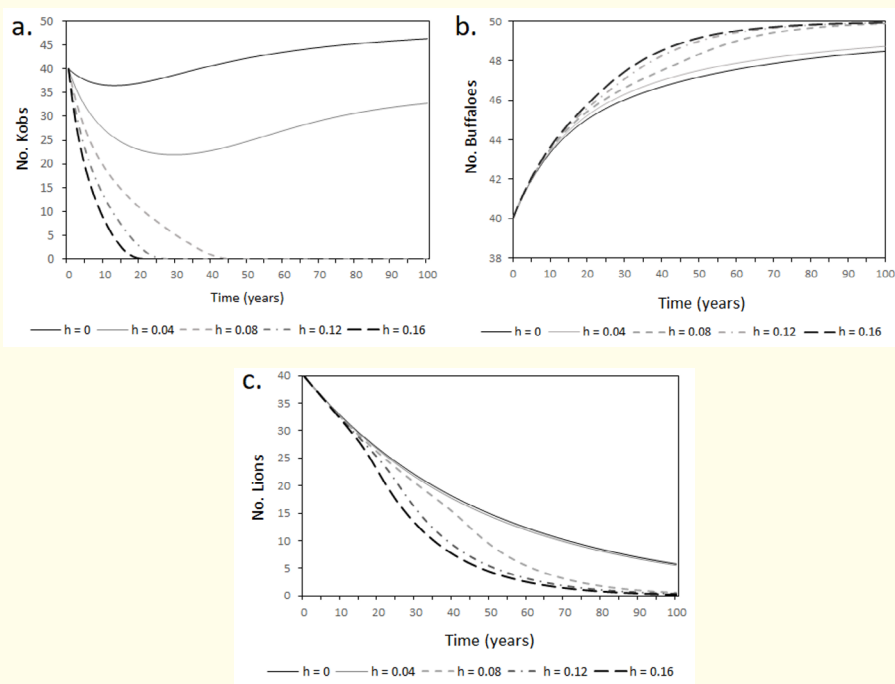


Figure 1: Results on the assessment of the effect of harvesting rate of the *kobus kob thomasi* on the dynamics of two-prey, one-predator ecosystem with prey X (*kobus kob thomasi*) in panel (a), prey Y (*Syncerus caffer*) in panel (b) and predator Z (*Panthera leo*) in panel (c).

From figure 2, we observe that co-existence of the three species over the 100-year period required that the capture rate of *kobus kob thomasi* by the predator be low as this benefits both this prey and the predator itself. Figure 2a shows that the population of *kobus kob thomasi* initially declined for all capture rates explored but was able to regain in the two scenarios with its lowest rates. The duration of the population declining phase was higher for higher capture rates and these also influenced the ultimate population sizes. Note that *kobus kob thomasi* species were extinct by around 20 years under their highest capture rate scenario.

On the other hand, the *Syncerus caffer* population increased in all the three scenarios explored but this occurred after an initial decline for its highest capture rate explored (Figure 2b). The ultimate population size was higher for lower capture rates and vice versa for higher rates. The population dynamics of the *Panthera leo* species under the scenario where the capture rate of *Syncerus caffer* was halved while doubling that of the *kobus kob thomasi* species (Figure 2c) were rather ecologically interesting. Capturing more *kobus kob thomasi* led to an initial increase in *Panthera leo* population and a substantial reduction in the prey populations. This increase lasted approximately 20 years and was followed by a sharp decline in the population, coinciding with the period when *kobus kob thomasi* were extinct. This decline continued and by 100 years, the predator was nearly extinct as well. In the other lower *kobus kob thomasi* capture rate scenarios, there was no initial boost in the predator population. It was the predator's overall slow decline in numbers that ensured their existence throughout the 100 years. Generally, lower capture rates of *kobus kob thomasi* yielded slightly higher numbers of the predator at any given time.

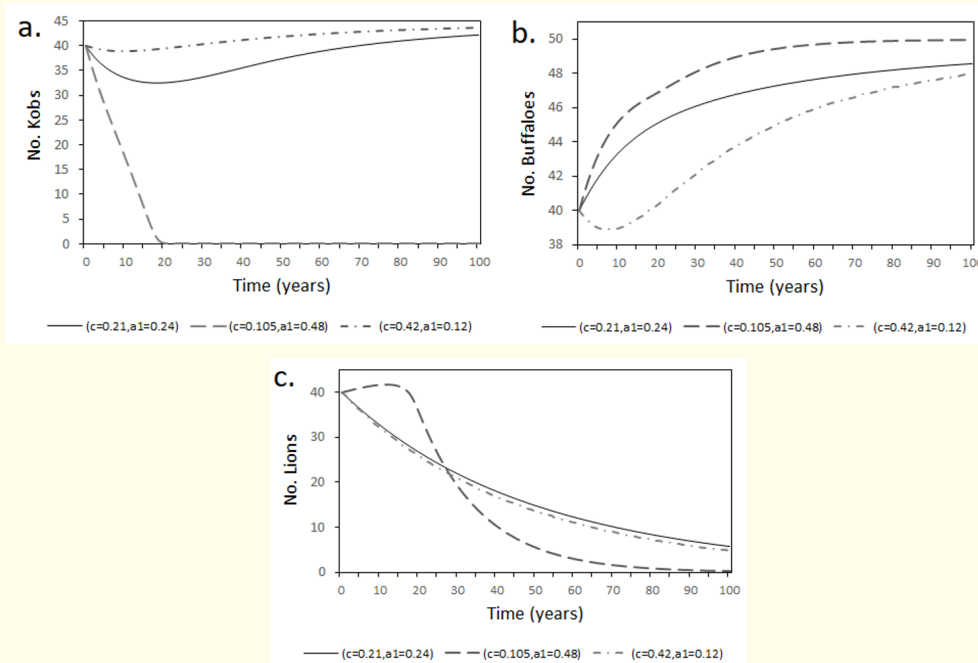


Figure 2: Results on the assessment of the effect of prey capture rates by the predator on the dynamics of two-prey, one-predator ecosystem with prey X (*kobus kob thomasi*) in panel (a), prey Y (*Syncerus caffer*) in panel (b) and predator Z (*Panthera leo*) in panel (c).

Figure 3 on the effect of interspecies competition reveals that, for all the explored parameter ranges and scenarios, the population of *kobus kob thomasi* initially declined and then increased towards the carrying capacity. The duration of the declining phase increased with an increase in the magnitude of the competition coefficients (Figure 3a). The population of *Syncerus caffer* increased throughout for the

two of the explored scenarios with the lower competition levels (Figure 3b). At the highest level of competition, its population trend was similar to that of the *kobus kob thomasi*, i.e. starting with a decline and then picking up afterwards. In all the three scenarios explored, the population of *Panthera leo* species was only marginally affected (Figure 3c) with all depicting a declining trend in population size over the 100 years' period. The only slight variation was in how fast the population declined, with lower competition rates leading to slightly slower declines.

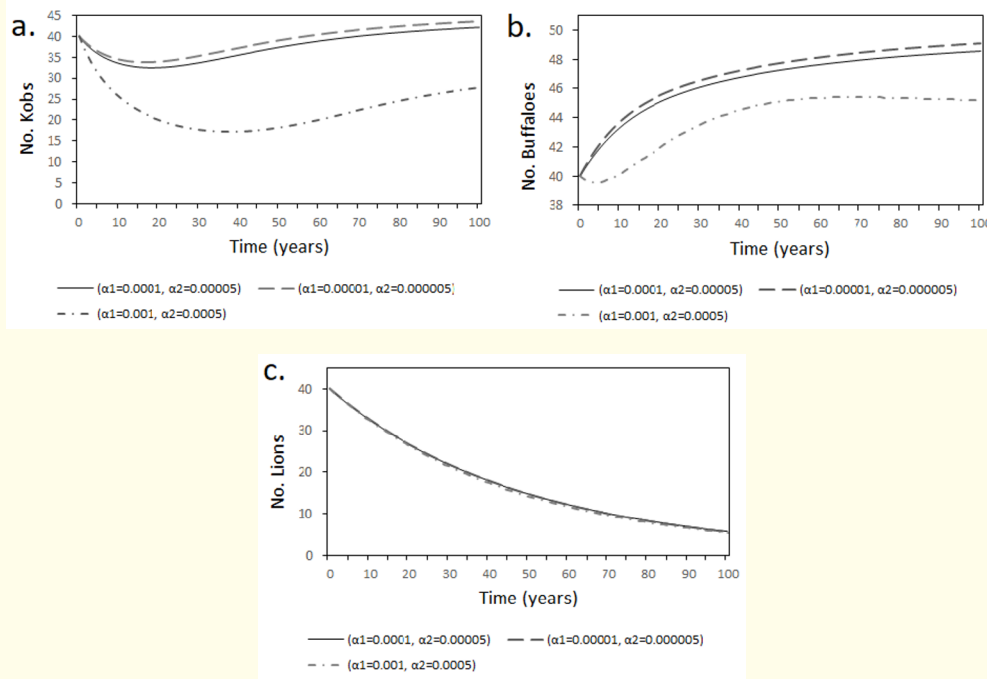


Figure 3: Results on the assessment of the effect of interspecies competition between *kobus kob thomasi* and *Syncerus caffer* on the dynamics of two-prey, one-predator ecosystem with prey X (*kobus kob thomasi*) in panel (a), prey Y (*Syncerus caffer*) in panel (b) and predator Z (*Panthera leo*) in panel (c).

Discussion and Conclusion

Predator-prey models play an important role in informing the management of renewable resources [44] and mathematical models have been demonstrated to support this effort [40]. As partly done in [7,15,30,31,34], here, a mathematical model of two prey-one predator system with functional response of Holling Type II, ratio-dependent response of a predator, and a constant effort harvesting (harvesting) of the prey and logistic growth of the prey is developed and analyzed both qualitatively and quantitatively.

Our analytical results and the numerical results in figure 1 show that the harvesting rate greatly affects the co-existence and stability of the system. For co-existence in the ecosystem, our findings reveal that it is necessary that the rate of human harvesting of the *kobus kob thomasi* does not exceed the intrinsic growth rate of the species. A critical *kobus kob thomasi* harvesting threshold by humans was found to exist. We found that any harvesting effort exceeding this threshold would lead to an ultimate extinction of not only the *kobus kob thomasi* species but also the *Panthera leo* and only the *Syncerus caffer* would thrive in such an ecosystem. In all the harvesting scenarios

explored, the population of *Syncerus caffer* species increased towards the system's carrying capacity throughout albeit at rates that increased with increasing harvesting rates of *kobus kob thomasi*. This is likely to be a consequence of a reduction in between-prey competition for resources due to a reduction in the number of competitors via human harvesting.

The critical harvesting threshold was numerically determined here to be around 0.04 per animal per year and a more precise estimate for this threshold can be determined by narrowing the gap between consecutive values explored. Note however that this estimate is dependent on the parameter values used and, if good data are available to facilitate reliable estimation of this threshold, it can be used to guide harvesting policy formulation. For example, Mayaka., *et al.* [40] used population dynamics models to compare alternative management strategies particularly illegal harvesting of *kobus kob kob* species and quantified the effect of this practice on the species' population dynamics. They concluded that monitoring and harvesting of populations are important aspects of wildlife management.

Our findings also indicate that the two preys can co-exist in the absence of the predator. This co-existence would be favored if the natural mortality rate of the predator is high and/or when its efficiency to convert the prey biomass into reproductivity is too low. Kesh., *et al.* [17] report similar results on this as well as on harvesting effect from their in-depth qualitative analysis study and also present all the steps of the qualitative analysis.

We found that the prey species, *kobus kob thomasi*, are a key determinant of the survivability of the *Panthera leo* species. This is demonstrated in figure 2 in which ecologically interesting dynamics were observed from assessing the effect of prey-capture rate. A potential initial growth in the *Panthera leo*'s population is observed in the scenario where default capture rate of *Syncerus caffer* is halved while doubling that of the *kobus kob thomasi* species. This initial increase turned out to be detrimental for the predator in a long run as one of the prey becomes extinct in a short time. This extinction seems to lead to starvation, death and ultimate extinction of the predators within the study as depict in their declining trend. Based on this, we can conclude that, for their continued survival, the predator needs not to only focus on the easy-to-catch prey but strike a balance between the two preys that ensures non-extinction of the prey. Continued survival of the prey is important for the predator's long-term survival. The predator should deploy a predation strategy that ensures non-extinction of the easy-to-catch prey. Moreover, these results are supported by the outcomes from our other numerical simulations (not shown) in which the predator population was shown to increase when the intrinsic growth rates of both prey was increased.

On the effect of competition between the two preys, higher competition coefficients lead to delayed increase in prey populations and consequently a faster decline in the predator populations as shown in figure 3. The long-term co-existence of the three species is heavily dependent upon the magnitude of the prey-competition coefficient as these are assumed (in our model) to be the only food sources for the predator.

In their study, Kesh., *et al.* [17] also analyzed a system involving one predator and two prey in which the prey species were competing and the predator's functional response is ratio-dependent. They showed that the introduction of a predator could either lead to co-existence of the three species or have some extinction of one the competing prey species and also predator under certain parametric conditions. These findings are in total agreement with those from this study.

Also, Kar and Chaudhuri [34] considered a two-prey one-predator harvesting model involving interference with predation, competition amongst and harvesting in both prey species (c.f. harvesting of only one prey in our study) while Kar [45] studied a prey-predator system by incorporating delay (in order to ensure that only mature preys were harvested), Holling Type II functional response harvesting of the prey. In their findings, they reported that as harvesting effort increased, the predator's population decreased. More importantly, if the harvesting effort was above their determined critical value, the dynamics of the system was asymptotically stable and the species goes to extinction. This showed that harvesting of the prey alone indirectly affected the population density of the predators and also played a crucial role in stabilizing the dynamics of the prey-predator systems. The findings from these studies are in close agreement with our findings and this validates our approaches to model building.

In summary, our assessment of the effect of various model parameters on the system dynamics revealed that the harvesting rate, interspecies competition and capture/predation rate all impacted the system dynamics albeit differently with some leading to extinction of some species in a long run. Yet, to a conservationist, the most interesting situation is that of the co-existence of all the three species and this is largely influenced by the human harvesting rate and the predator capture rate of *kobus kob thomasi* among our explored scenarios. The key limitation of this study is the absence of field data to reliably parametrize the model and this would require conducting field and experimental studies that are in most cases constrained by logistics. With better data, such findings would inform policy decisions on nature and environmental conservation at the interface of human-livestock-wildlife.

We conclude that the model has helped to investigate the various conditions under which the three species can co-exist; the numerical solutions have also demonstrated the need for controlled harvesting if co-existence of the three species in the ecosystem is to be attained. Given the study limitations such the absence of adequate data, we can assert that our findings are rather preliminary. Nonetheless, and most importantly, this study demonstrates that mathematical models can play a significant role in tackling complex system dynamics. Particularly, we were able to identify the most impactful parameters. Such information would guide prioritization of future research, in this case, making decisions on which studies on parameter estimation to conduct first. In addition, the required level of precision in estimating those parameters would also depend on level of impact that the parameter has on the dynamics of the system.

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